In developing the approach featured in this book, I have deconstructed the essential teaching points of the numeracy strategies listed above and teased out a structured and logical sequence for teaching and learning. The resulting series of teaching activities provides for a systematic and cumulative progression in very small incremental steps, frequently reinforced. In this way, pupils can be introduced to a single new idea at every step, without being prematurely exposed to problems beyond their level of understanding. Pupils are therefore able to experience success while developing their mathematical understanding.

I make no attempt to devise a formal or prescriptive teaching programme. Instead, I have tried to record, in as detailed and accessible a manner as I can manage, ideas that I have found to be successful with my own pupils in combating a range of common misconceptions and difficulties. The work is predominantly practical and oral. I prefer to avoid worksheets because I suspect that having pupils work through pages of written examples will only result in the pupils reinforcing the same inefficient strategies and bad habits that have contributed to their lack of progress.

I strongly believe that the teacher is each pupil's most important resource and that the teaching focus should always be on teaching for understanding by developing logical thinking and mathematically sound cognitive models.

The activities and ideas in this book are all ready to use with a minimum of preparation. The only equipment needed is what is commonly available in the classroom or easily purchased: dominoes, dice, playing cards and number cards, Cuisenaire rods and base-ten blocks, paper and pencil. Various other resources, including game boards and information about Cuisenaire rods, can be found and printed off from the accompanying Online Resources on the book's website.

The contents of the book are organised into five parts:

- Part I – Stop counting in ones.
- Part II – The bridging technique.
- Part III – The area model of multiplication and division.
- Part IV – Reasoning strategies.
- Part V – Games collection.

How to use this book

The first four parts of the book are set out sequentially, with teaching activities presented in later parts designed to rest on numerical understanding developed at an earlier stage. For example, in order to be able to use the bridging technique (Part II) or to learn efficient strategies to solve multiplication and division problems (Part III), children must have already progressed beyond the stage of relying on their fingers to find simple addition and subtraction answers (Part I).

Each chapter begins with an overview of the topic that puts the teaching points into context. Each chapter also includes a summary of the individual steps that are later expanded in detail.

The ideas in the first four parts of the book are structured into a carefully managed sequence of teaching activities designed to be teacher-led. If you are looking for activities and games for parents who are willing and able to support their own children at home, you can find plenty of suitable ideas in the three chapters devoted to putting in place all the necessary pre-skills (Chapters 1, 2 and 5).

Part I addresses one of the most common obstacles to struggling pupils' progress in numeracy, namely their tendency to rely on counting in ones. This topic is dealt with in a great deal of detail in my previous book *The Dyscalculia Toolkit* and in the two *Exploring Numbers* titles of my ebooks, which is why Part I of this book is relatively short with a focus on practice, rather than teaching, of number components. Once pupils have been taught how to free themselves from the 'counting trap' they will still need plenty of opportunities to consolidate what they learned, for which purpose a whole range of teaching games and puzzles are provided for practice. A table listing the recommended games against their main teaching points can be found at the end of Chapter 1.

Although it may take a long time for children to completely outgrow their inclination to count in ones, this need not hold them back from beginning to work on the more challenging ideas set out in Part II, at the same time.

Part II contains a detailed step-by-step guide to teaching the bridging technique for both addition and subtraction. Following an analysis of the requisite pre-skills together with suggested activities for teaching these pre-skills to pupils for whom the concepts are not yet secure, two further chapters are devoted to the teaching of bridging through 10, and then through multiples of 10. These chapters advocate the teaching of subtraction as complementary addition, and explore how pupils can move from the concrete stage to the purely abstract stages of mental calculation.

Part III contains a detailed step-by-step guide to teaching the area model for multiplication and division. The first two chapters address both operations together and include an analysis of the requisite pre-skills complete with suggested activities for teaching these pre-skills to pupils for whom the concepts are not yet secure. Two further chapters are devoted to the teaching steps through which pupils can learn to manage the transition from the concrete stage to the abstract stage of understanding the standard written algorithms.

Part IV focuses on reasoning strategies. Pupils with difficulties in maths are rarely flexible in their thinking and must be explicitly taught how to use the few facts they know to derive new facts.

Part V contains all the games recommended in the book, collected together for convenience and ease of access.

The Online Resources accompanying this book contain more than 30 pages of printable resources. They include an appendix about Cuisenaire rods and other concrete teaching equipment, together with various resources such as all the games boards and digit cards required for the games and puzzles.

Access to the four videos made to accompany this new edition is via the OR ↻.

Dyslexia

A dyslexic pupil might show many of the same indicators as those mentioned above, because it is thought that at least half of all dyslexics also have difficulties with maths. Outside the maths classroom, you might suspect that pupils are dyslexic if they read and write much less willingly and fluently than you might expect, if they read and reread written material with little comprehension and if their spelling is particularly weak, inconsistent or bizarre. Dyslexic learners show much greater ability and understanding when speaking than you could ever guess from looking at the scrappy and minimal amount of written work they produce. Other indicators are memory weaknesses, problems with processing auditory information, and difficulties with planning and organisation.

Dyspraxia

A typical dyspraxic pupil does not seem to have the same long-term memory problems as a dyslexic and so might be able to remember times tables facts with ease. Dyspraxia, also known as DCD (developmental coordination disorder), mainly affects motor control, which results in pupils being clumsy and uncoordinated, poor at planning and organisation, and unsuccessful at subjects like PE and sports that require balance and coordination. Dyspraxic pupils cannot process sensory information properly and are therefore forever tripping and falling, dropping and breaking things, and mislaying their belongings. In the maths classroom, dyspraxic pupils have particular difficulty handling equipment such as a ruler, a protractor or a set of compasses, and their written work is likely to be very messy and difficult to decipher.

Diagnosis

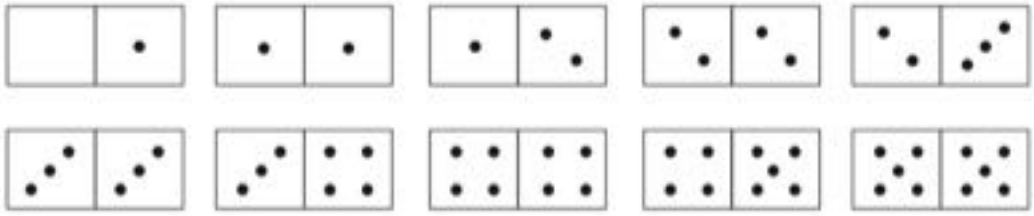
A quick and informal way of identifying pupils who need extra help, or further assessment, is to: (a) find whether pupils have difficulties counting backwards, (b) discover which pupils cannot remember times tables reliably, and (c) notice which pupils have no calculation strategies beyond counting in ones. A less subjective identification can be achieved by using Brian Butterworth's computer-based Dyscalculia Screener, published under the nferNelson imprint and obtainable through GL Assessment. The Screener, which is based on Professor Butterworth's neuroscientific research, can be administered to several pupils at once and produces a profile of each pupil that can provide evidence (or an absence of evidence) of dyscalculia. A formal diagnosis of dyscalculia can only be given by a qualified educational psychologist after a thorough assessment.

How to help pupils who have difficulties with numeracy

The following principles, exemplified in this book, summarise my approach to teaching learners with dyscalculia:

- Start with concrete materials, making sure that the equipment you use is mathematically sound and is robust enough to model a wide range of numeracy topics. In my opinion, the best concrete resource is a collection of base-ten materials, such as Cuisenaire rods, supplemented by Dienes blocks.

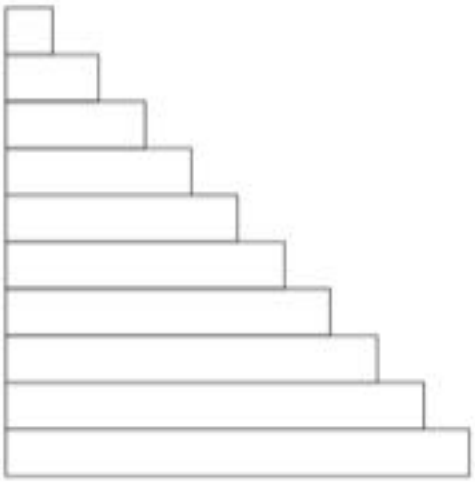
and then all five of the odd numbers. Together, these ten cards comprise one full set of domino cards. Note that each domino card shows only the key fact for one of the numbers up to 10. By contrast, a full set of European dominoes has 28 tiles or stones, from double-zero to double-six, with all the combinations in between.



A set of domino cards, showing the key component fact for each of the numbers up to 10.

Cuisenaire rods

Although dice patterns and domino spot patterns are good for promoting visual recognition of component facts, these patterns are all built from discrete units. Over-reliance on such patterns will inevitably encourage pupils to continue thinking about numbers as composed of quantities of ones, which is precisely what we are trying to discourage. Cuisenaire rods, on the other hand, show each of the numbers 1 to 10 as a single unit. For example, the black rod representing the number 7 can not only be measured in ones to demonstrate that it is equivalent to seven ones, but is also a discrete unit in its own right, i.e. one seven. This is why Cuisenaire rods are such a powerful tool in the fight against an over-reliance on counting in ones.



I find Cuisenaire rods to be an invaluable resource, responsible for many of those 'aha!' moments when suddenly things seem to click into place in a learner's mind. As with all other concrete teaching materials, rods should never be used to find the answer to a calculation mechanically, but, instead, be used as tools for modelling arithmetic thinking.

A short introduction to concrete manipulative materials, including Cuisenaire rods, can be found on the OR ↻.

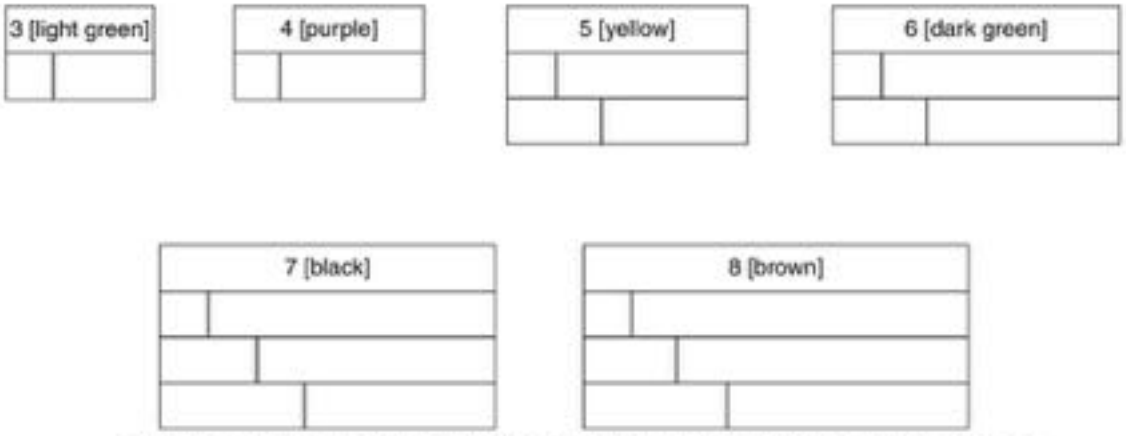
Activity

Build numbers concretely out of two different components

For children who persist in counting in ones, I strongly recommend that you give them hands-on experience with concrete materials that are continuous (as opposed to discrete objects such as counters). Here is one example of a particularly useful hands-on activity using Cuisenaire rods, but you can find many more in *The Dyscalculia Toolkit* and *Exploring Numbers Through Cuisenaire Rods*.

This activity is designed to target pupils who are still counting up on their fingers while adding single-digit numbers. Using a set of Cuisenaire rods, ask the child to build the numbers up to 8 out of two different components, i.e. two rods of different colours placed end to end, in as many ways as they can. This activity forces the child to notice that there is only one way of building the numbers 3 and 4 (the doubles fact for 4 would need two rods of the same colour), only two possible alternatives for building 5 or 6, and only three possible ways of building 7 or 8 from two components. Can the pupil now predict, and then check, how many different ways there are of building 9 or 10 out of pairs of different components?

Discovering how very few combinations exist often reassures pupils who might otherwise believe that they are expected to memorise an unfeasible number of component facts.



Building small numbers out of two different rods highlights how few possibilities there are. This reassures pupils that learning about components is not as hard as they might expect.

This is a good activity to explore, or revise, in preparation for tackling the component Su Doku puzzles (provided on the OR ⟳). For the first few puzzles, allow the rods to stay within view so that pupils can use them for reference.

Many of the practical teaching suggestions in Parts II and III of this book make use of Cuisenaire rods, so do make sure your pupils are given ample opportunities to become familiar with the colours and sizes of the rods before beginning to use them for the hands-on maths activities that begin in Part II.

Instructions for half a dozen Cuisenaire rods games that can help children stop counting in ones are provided in the Games Collection in Part V of this book.

Component games: Main teaching point

Pelmanism	Numbers 1–10 built from key component facts
3-in-a-row Key Components	Numbers 1–10 split into key components
Domino Cards Solitaire	Numbers 1–10 built from key component facts
Make a 1–10 Sequence	Adding small quantities
Pick-Up Addition Race	Adding small quantities
Pick-Up Subtraction Race	Subtraction as finding the difference
Round the Spot	Odd or even; adding small quantities
Centenniel	Adding small quantities
Triples Addition	Adding small quantities
Odds or Evens	Odd or even; adding small quantities
Stuck in the Mud	Adding small quantities; keeping a running total
Who Has the Last Word?	How to build a number in different ways
The 3-Component Challenge	Build a number out of three components
Cherry Picking	Build a number out of several components
Rods in Blocks	Addition; bridging through 5
Descent	Subtracting and decomposing 1-digit numbers
Full House	Complements (number bonds) to 10
Tens Away	10 built out of a pair of components
Eleven Up	11 built out of a pair of components
Clock the Twelves	12 built out of a pair of components
Thirteens & Fifteens	13 and 15 built out of pairs of components
Fifteen in a Suit	15 built out of smaller components
Standing Aces	Adding pairs of numbers up to a total of 20
Pyramid Solitaire	13 built out of a pair of components
Prisoners	11, 12 and 13 built out of pairs of components
Pontoon	Adding small numbers up to a total of 21
Zero Blackjack	Adding and subtracting small numbers
Shut the Box	Adding small numbers to match a given target
Banking Tens	Complements (number bonds) to 10
Marching On	Complements; step-counting in 10s
Conjure the Number	Adding and subtracting small numbers
Subtract From 15	Subtraction from a teen number, e.g. from 15

(Continued)

Minimise the Difference	Subtracting 1-digit numbers
Maximise the Difference	Subtracting 1-digit numbers
Plus or Minus	Addition or subtraction up to a total of 20 or 30
A Close Call	Mentally adding two 2-digit numbers
Magic Squares & Missing Number Puzzles	Components in addition and subtraction
Component Su Doku Puzzles	Components in addition and subtraction

A second table, in which all 45 of the recommended games in this book are listed against the equipment required, can be found in Part V.

A ten-minute video about Part I, including short demonstrations of the two practical activities in this chapter and some of the recommended games, can be accessed via the OR ↻.

CHAPTER TWO

Pre-skills for learning the bridging technique

Overview

Bridging through 10, or through multiples of 10, is the single most useful mental calculation strategy that pupils can learn.

Bridging through 10 is the technique by which addition is performed as a linear movement on a number line. Pupils must be introduced carefully to the idea of a number line and shown how it differs from a number track. The number line on which bridging is modelled will, at first, be an actual straight edge or a line drawn on paper, but with sufficient practice pupils can learn to visualise an imaginary number line on which to perform mental addition. Crucially, movement along the line is not performed one step at a time, but in just two jumps. The number 10, a significant number in our decimal number system, acts as a stepping stone between the two jumps. The other important and notable feature of bridging through 10 is that work can always be performed from left to right, in the forward direction.

Before learning the bridging technique, pupils must first have mastered certain pre-skills. Ideally, the necessary pre-skills will have been acquired at primary school, but for older learners who still need to work on these skills and concepts, teaching suggestions can be found in this chapter. If your pupils have parents who are willing and able to help with practice at home, this chapter is a good source of ideas for suitable activities and games. For learners who have already thoroughly understood and internalised the necessary pre-skills, turn to Chapters 3 and 4 for a step-by-step guide to teaching the bridging technique.

An 11-minute video about the three bridging chapters in Part II can be accessed via the OR ⟳.

- Clear the Deck solitaire game.*
- Component Su Doku puzzles (see Chapter 1 and the OR).

3. The connection between addition and subtraction

Addition and subtraction should be taught alongside each other in ways that reinforce the underlying connection between them. Pupils should be shown how both addition and subtraction can be used to describe or record a single relationship, from different points of view. I sometimes use the analogy of a brother and sister: the girl says she has a brother, the boy claims a sister, but both are describing the same relationship, each from a different perspective.

Some suggested teaching activities are:

- Missing rods from Cuisenaire rods sandwiches.*
- Equations made of Cuisenaire rods to build, manipulate and read.*
- Simple equations to record on paper as both additions and as subtractions.*
- Hidden quantity scenarios recorded in various ways.*
- Word problems to be made up by pupils to match a given number fact.*
- Post-It Note Subtraction game.*
- An informal triangular notation of the triad of related facts. For example, diagonal lines to record the fact that 8 can be made of 2 and 6. In the illustration, notice the plus sign between the two numbers below the diagonal lines. The operation sign is included so that there is no chance of mistaking the two components for a single 2-digit number, but this should not stop pupils from viewing and reading the trio of numbers as a subtraction fact as well as an addition fact.

An informal triad notation shows a family of related addition/subtraction facts.

4. The difference between a number track and a number line

These two terms are not interchangeable: they describe two different concepts. Pupils who confuse the two often produce answers that are one more or less than the correct solution.

A number track is a model of the number system that allocates a defined space, or area, to each number. Pupils' first encounter with a concrete number track at school might be an activity that involves arranging counters in a line and touching each one as they count to find the total amount. Before school age, many children will already have become familiar with diagrammatic number tracks in the form of board games, such as Snakes & Ladders. However, the number track is not a model that is exclusively associated with young children and small numbers, as can be seen by the 100-square